Abstract: Let $M$ be a compact Riemannian manifold. It was proved by Weyl that number of Laplacian eigenvalues less than $T$, is asymptotic to $C(M)T^{\text{dim}(M)/2}$, where $C(M)$ is the product of the volume of $M$, volume of the unit ball and $(2\pi)^{-\text{dim}(M)}$. Let $\Gamma$ be an arithmetic subgroup of $SL_2(\mathbb{Z})$ and let $\mathbb{H}^2$ be an upper-half plane. When $M = \Gamma \backslash \mathbb{H}^2$, Weyl’s asymptotic holds true for the discrete spectrum of Laplacian. It was proved by Selberg, who used his celebrated trace formula.

Let $G$ be a semisimple algebraic group of Adjoint and split type over $\mathbb{Q}$. Let $G(\mathbb{R})$ be the set of $\mathbb{R}$-points of $G$. For simplicity of this exposition let us assume that $\Gamma \subset G(\mathbb{R})$ be an torsion free arithmetic subgroup. Let $K_\infty$ be the maximal compact subgroup. Let $L^2(\Gamma \backslash G(\mathbb{R}))$ be space of square integrable $\Gamma$ invariant functions on $G(\mathbb{R})$. Let $L^2_{\text{cusp}}(\Gamma \backslash G(\mathbb{R}))$ be the cuspidal subspace. Let $M = \Gamma \backslash G(\mathbb{R})/K_\infty$ be a locally symmetric space. Suppose $d = \text{dim}(\Gamma \backslash G/K_\infty)$. Then it was proved by Lindenstrauss and Venkatesh, that number of spherical, i.e. bi-$K_\infty$ invariant cuspidal Laplacian eigenfunctions, whose eigenvalues are less than $T$ is asymptotic to $C(M)T^{\text{dim}(M)/2}$, where $C(M)$ is the same constant as above.

We are going to prove the same Weyl’s asymptotic estimates for $K_\infty$-finite cusp forms for the above space.