Abstract: Lie theory is a systematic framework for understanding continuous symmetry. It is therefore crucial to the study of geometric objects carrying large symmetry groups, and to the process of taking quotients by such groups. This is particularly apparent within symplectic geometry, the mathematical abstraction of Noetherian classical mechanics. One is often dealing with a Hamiltonian symplectic variety, i.e. a symplectic algebraic variety $X$ endowed with a structure-preserving action of an algebraic group $G$. Marsden and Weinstein defined the first notion of a "symplectic quotient" of $X$ by $G$ in 1974, and many variants of this construction have emerged in the ensuing years. Symplectic quotient constructions are now crucial to research at the interface of algebraic geometry, differential geometry, geometric representation theory, and mathematical physics.

I will give a non-technical overview of the themes mentioned above. Some emphasis will be placed on a new, purely Lie-theoretic approach to taking quotients in symplectic geometry. It generalizes several of the quotient constructions developed over the last 50 years, and is the result of joint work with Maxence Mayrand.