Abstract: A theorem of Northcott states that a rational function of degree at least 2 with rational coefficients has only finitely many rational preperiodic points. In a different direction, more recent work of Baker and DeMarco says that if two rational functions with complex coefficients do not have the exact same set of complex preperiodic points, then they have only finitely many complex preperiodic points in common.

In both settings, it has been conjectured that the (necessarily finite) number of preperiodic points should be uniformly bounded above by a constant depending only on the degrees of the rational functions involved. However, there is hardly any evidence to suggest just how large these uniform bounds should be.

In this talk, I will discuss the first known nontrivial lower bounds for both of these conjectured uniform upper bounds. These lower bounds are obtained by proving the existence of polynomials in each degree with interesting dynamical properties, including having many rational preperiodic points. This is joint work with Trevor Hyde.