Singular stochastic PDE’s and non-uniqueness in mathematical physics and fluid mechanics

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Abstract: In this talk, I would like to explain recent developments in PDE’s of my interest. First, many PDE’s that were derived and analyzed by physicists consisted of random force, specifically space-time white noise. Examples include Kardar-Parisi-Zhang equation or Phi4 model from quantum field theory. The roughness of such noise led to a lack of sufficient spatial regularity for the solution and therefore the non-linear terms therein to be ill-defined. The recent inventions of the theory of regularity structures by Hairer and the theory of para-controlled distributions by Gubinelli, Imkeller, and Perkowski nonetheless allow one to obtain some solution theory, and their contributions are considered to be exceptionally important in recent advances in mathematics. Second, the convex integration technique, originally accredited to the work of Nash in geometry in 1954, was recently extended significantly and adapted to equations of fluids. It has led to multiple breakthroughs, e.g., the resolution of Onsager’s conjecture about Euler equations from 1949, Taylor’s conjecture about MHD from 1974, non-uniqueness of the weak solution to the 3D Navier-Stokes equations which settled Serrin’s conjecture from 1963. After describing these developments, I would like to explain more recent developments, as well as open problems that connect these two techniques.