The Alexander Polynomial and Gordian Distance

Abstract: We call a knot $K$ a complete Alexander neighbor if every possible Alexander polynomial is realized by a knot one crossing change away from $K$. It is unknown whether there exists a complete Alexander neighbor with nontrivial Alexander polynomial. I will discuss how to eliminate infinite families of knots with nontrivial Alexander polynomial from having this property and possible strategies for unresolved cases. I will also discuss how a related condition on determinants of knots one crossing change away from unknotting number one gives an obstruction to unknotting number one. This obstruction appears similar to an obstruction introduced by Lickorish, but Lickorish’s obstruction does not subsume the obstruction coming from the condition on determinants.