Abstract: There is an idea in number theory that if two objects are congruent modulo a prime $p$, then the congruence can also be seen for the special values of $L$-functions attached to the objects. Here is a context explicating this idea: Suppose $f$ and $f'$ are holomorphic cuspidal eigenforms of weight $k$ and level $N$, and suppose $f$ is congruent to $f'$ modulo $p$; suppose $g$ is another cuspidal eigenform of weight $l$; if the difference $k - l$ is large then the Rankin-Selberg $L$-function $L(s, f \times g)$ has enough critical points; same for $L(s, f' \times g)$; one expects then that there is a congruence modulo $p$ between the algebraic parts of $L(m, f \times g)$ and $L(m, f' \times g)$ for any critical point $m$. In this talk, after elaborating on this idea, I will describe the results of some computational experiments where one sees such congruences for ratios of critical values for Rankin-Selberg $L$-functions. In the second half of the talk I will sketch a framework involving Eisenstein cohomology for $GL(4)$ over $\mathbb{Q}$ which will permit us to prove such congruences. This is joint work with my student P. Narayanan.