Abstract: Interface problems are ubiquitous. They arise widely in many applications in science and engineering. For mathematicians, partial differential equations (PDE) are often used to model such interface problems. Solutions to these PDE interface problems often involve kinks, singularities, discontinuities, and other non-smooth behaviors. The immersed finite element method is a class of numerical methods for solving PDE interface problems on unfitted meshes.

In this talk, we introduce some recent advances in immersed finite element methods. First, we will briefly review the main idea of immersed finite element methods. We will talk about how this immersed FEM idea can be fitted into some nontraditional finite element frameworks, such as nonconforming finite element, discontinuous Galerkin, and weak Galerkin frameworks. All of these numerical methods are proved to yield optimal a priori error estimates. Next, we will present a residual-based a posteriori error estimation for IFEM. Our error estimator is shown to be globally reliable and locally efficient, with the reliability and efficiency constants independent of the interface location. This error estimator can provide accurate assessment of the current numerical approximation and can also be used as a guidance for adaptive mesh refinement for immersed finite element methods. Numerical results are provided to demonstrate the features of these immersed finite element methods.