**Max Dehn and Helmut Kneser in 1929**

**or**

**What went wrong with Dehn’s Lemma?**

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Max Dehn “proved” his famous lemma in a well-known paper on 3-manifolds from 1910. The lemma claims that certain immersed discs in a 3-manifold can be replaced by embedded discs. Dehn’s argument used fairly complicated cut-and-paste methods. As is well known, almost two decades later Helmuth Kneser found a gap in the proof. This led to an interesting correspondence that will be described in this talk, mainly be looking at the types of examples that Dehn and Kneser discussed. B. L. van der Waerden soon afterward gave a report on research in higher-dimensional topology in which he curiously noted that the gap in Dehn’s proof had long been known. It seems likely that Dehn afterward conjectured that the lemma was false. He produced an unrealizable Dehn diagram, reproduced by the Norwegian topologistIngebrigt Johansson. However, in 1935 Johansson gave necessary and sufficient conditions for a Dehn diagram to be realizable, i.e. correspond to the singularity set for an immersed disc. This showed that Dehn’s example did not lead to a counterexample after all. It took another two decades before Christos Papakyriakopoulos finally proved Dehn’s lemma.