Title: Optimization Problem for Klein-Gordon Equation

Abstract: We consider a damped Klein-Gordon equation with a variable diffusion coefficient of the form

$$u_{tt}(t,x) + \alpha u_t(t,x) - \nabla(\beta(x)\nabla u(t,x)) + \delta g(u(t,x)) = f(t,x)$$

where the diffusion coefficient $\beta(x)$ is Lipschitz continuous. This Klein-Gordon equation is known as one of the nonlinear wave equations arising in relativistic quantum mechanics. The goal is to derive necessary conditions for the optimal set of parameters minimizing the objective function J. First, we show that the solution map is continuous. Then the solution map is shown to be weakly Gateaux differentiable on the admissible set P, implying the Gateaux differentiability of the objective function. Finally we study the Frechet differentiability of J and optimal parameters for these problems. Unlike the sine-Gordon equation, which has a bounded nonlinear term, Klein-Gordon equation requires stronger assumptions on the initial data.