Master's Comprehensive Exam
Complex Variables
August 1995

1. (10 pts.) Compute $\mathrm{Log}(i)$, where Log represents the Principal branch of the logarithm.

2. (30 pts.) Let $f(z) = z \cdot \overline{z}$.

   (a) Where is $f(z)$ continuous?

   (b) Where is $f(z)$ differentiable and what is its derivative at points where it is differentiable?

   (c) Where is $f(z)$ analytic?

3. (20 pts.) In each case, find the residue of the function at 0 and indicate whether 0 is a removable singular point, an essential point or a pole (in which case, give the order of the pole).

   (a) $\dfrac{e^z - 1}{z}$;

   (b) $\dfrac{(1+z)\sin(z)}{z^2}$

4. (15 pts.) Using the Residue Theorem, compute the following integral:

$$\int_0^{+\infty} \frac{\mathrm{d}x}{1+x^2}.$$

5. (10 pts.) Find the linear fractional transformation T which maps the points $-1, 0, 1$ onto $-i, 1, i$ respectively.

6. (15 pts.) Find the image of the unit disk under the following linear fractional transformation T:

$$T(z) = \frac{i-z}{i+z}$$