Master's Exam
Topology
August 1995

1. (a) If X is a space and $A \subseteq X$, define the *closure* of A in X.
   (b) Let Y be a subspace of X; let A be a subset of Y; let $\bar{A}$ be the closure of A in X. Prove that the closure of A in Y equals $\bar{A} \cap Y$.

2. Let $I = [0, 1]$. Let I^ω be the standard cube in $\mathbb{R}^\omega$ in the product topology.

   (a) Prove that there is a continuous function $f : I^\omega \to [0, 1]$ that takes the value 0 at the origin $(0, 0, ...)$ and that takes the value 0.9 on the whole subcube $[0.9, 1]^\omega$
   (b) Take some f that satisfies part (a) and assume that it never takes a value < 0 or ≥ 1. Prove that there is an $a < 1$ such that the image of f lies in $[0, a]$.
   (c) In the situation of part (b), prove that $a/2$ is in the image of f.

   Be sure to verify the hypothesis of any theorems or lemmas you use.

3. Give an example, with proof, showing that the quotient of a Hausdorff space need not be Hausdorff.

4. (a) Define the terms *Lindelöf* and *sequentially compact.*
   (b) Prove that a space that's Lindelöf and sequentially compact must be compact.