MASTER'S EXAM - TOPOLOGY - AUGUST 1998

1. Let X and Y be topological spaces. Let $A \subseteq X$ and $B \subseteq Y$. For each of the following inclusions give either a proof or a counterexample.
   (a) $\overline{A \times B} \subseteq \overline{A} \times \overline{B}$
   (b) $\overline{A} \times \overline{B} \subseteq \overline{A \times B}$
2. Let A be a subset of the topological space X. Assume that $\emptyset \neq A \neq X$. Let $B = X - A$. Let $Y = \{a, b\}$. Define $f : X \to Y$ by $f(x) = a$ if $x \in A$ and $f(x) = b$ if $x \in B$. Give Y the quotient topology induced by f. Prove that Y is Hausdorff if and only if the pair A, B is a separation of X.
3. Prove that every compact Hausdorff space is normal.
4. State the Urysohn Lemma. State the Tietze Extension Theorem. Prove that if the Tietze Extension Theorem is assumed to be true, then the Urysohn Lemma is true.
5. Let X be a space which satisfies the second axiom of countability. Let A be a subspace of X. Prove that A satisfies the second axiom of countability.