Master's Exam
Topology
June 1993

1. (a) Give the definition of a regular topological space.
   (b) Prove that every subspace of a regular space is regular.

2. Let X_α be a collection of spaces indexed by $\alpha \in J$. Let $\alpha_0 \in J$. Prove that the projection map
$$\pi_{\alpha_0} : \Pi_{\alpha \in J} X_\alpha \to X_{\alpha_0}$$
is a quotient map.

3. Let X be a topological space. A subspace $A \subseteq X$ is *locally closed* if any $x \in A$ has an open neighborhood U in X such that $U \cap A$ is closed in U (though $U \cap A$ is not necessarily closed in X). Prove that A is locally closed if and only if there is an open set $V \subseteq X$ and a closed set $C \subseteq X$ such that $A = V \cap C$.

4. Prove the *tube lemma*: consider the product space $X \times Y$, where Y is compact. Let $x_0 \in X$. Let N be an open set of $X \times Y$ containing the slice $\{x_0\} \times Y$. Then there is an open set $W \subseteq X$ containing x_0 such that N contains the tube $W \times Y$.