Master's Comprehensive Exam
Complex Variables
May 1995

1. (10 pts.) Compute i^i.

2. (10 pts.) Compute $\int_C z^2 \mathrm{d}z$, where C is the line segment from the origin to the point $(1+i)$ in the complex plane.

3. (20 pts.) In each case, find the residue of the function at 0 and indicate whether 0 is a removable singular point, an essential point or a pole (in which case, give the order of the pole).

   (a) $e^{1/z}$;

   (b) $\dfrac{\cos\ z - 1}{z^2}$.

4. (15 pts.) Using the Residue Theorem, compute the following integral:

$$\int_0^{+\infty} \frac{\mathrm{d}x}{1+x^2}.$$

5. (15 pts.) Find the Laurent series expansions in the domain $0 < |z| < 1$ for the function

$$f(z) = \frac{1}{z^2(1-z)}$$

6. (30 pts.) Let $u(x,y) = x^2 - y^2 + x$ where x and y are real variables.

   (a) Show that $u(x,y)$ is harmonic in the plane.

   (b) Find all the harmonic conjugates of $u(x,y)$.

   (c) Find an analytic function f such that $\mathrm{Re}(f) = u(x,y)$.