MASTERS EXAM - TOPOLOGY - MAY 1995

1. Let X be a topological space having the property that one-point sets are closed in X. Let $A \subseteq X$, and let A' be the set of all limit points of A in X. Prove that A' is closed in X.

2. Let (X, d) be a metric space. Prove that the function $d : X \times X \to \mathbf{R}$ is continuous.

3. (a) Prove that every path-connected space is connected.

   (b) Is the converse true? If it is true, then give a proof. If it is not true, then briefly describe a counterexample (without proof).

4. Let X be a second countable space. Prove that X is separable and Lindelöf.

5. Prove that every compact Hausdorff space is normal.