MASTER'S EXAM: GENERAL TOPOLOGY

June 1997

1. Let $X = \{a, b, c\}$ with the topology $\{X, \emptyset, \{a\}, \{b, c\}\}$. Let $A = \{a, b\}$. Find the interior, closure, set of all limit points, and boundary of A in X.

2. Let $f : X \to Y$ be a function.

   (a) Prove that if f is continuous, then for every sequence $\{x_n\}$ such that $x_n \to x$ in X one has $f(x_n) \to f(x)$ in Y.

   (b) Prove that if X satisfies the first axiom of countability, then the converse holds.

3. Prove that every infinite subset of a compact space has a limit point.

4. Prove that every metrizable space is normal.

5. Consider the circle S^1, the line $\mathbf{R}$, the non-negative ray $[0, +\infty)$, and the non-positive ray $(-\infty, 0]$. For each pair of these spaces determine whether or not they are homeomorphic. Give brief proofs of your answers.