MASTERS COMPREHENSIVE EXAM—COMPLEX VARIABLES—January 1995

1. Find the real and imaginary parts of $(-\sqrt{12}-2i)^6$.

2. In each case, find the residue of the function at 0, and indicate whether 0 is a removable singular point, an essential singular point, or a pole (in which case give the order of the pole).

   a) $z\sin\frac{1}{z}$

   b) $\dfrac{1}{z^3e^{2z}}$

   c) $\dfrac{1-\cos z}{z^2}$

3. In this problem we write γ for the contour defined by $\gamma(t)=e^{it}$ on $[0,2\pi]$. Evaluate each integral, with explanation.

   a) $\int_\gamma \sin z^3\,dz$

   b) $\int_\gamma z^{-2}e^{(z-1)^2}\,dz$

4. For fixed $r>0$ we write $D=\{z\colon 0<|z|<r\}$.

   a) If f defined by $f(z)=1/z$, find the image of D under f.

   b) If g is defined by $g(z)=e^{1/z}$, find the image of D under g.

5. Suppose u is a real-valued function on the complex plane $\mathbf{C}$.

   a) If also v is a real-valued function on $\mathbf{C}$, state the Cauchy-Riemann equations for (u,v).

   b) Recall that a real-valued function v on $\mathbf{C}$ is said to be a harmonic conjugate of u if $u+iv$ is analytic on $\mathbf{C}$. Use the Cauchy-Riemann equations to show that any two harmonic conjugates of u differ by a constant on $\mathbf{C}$.