Master's Comprehensive Exam
Complex Variables
January 1997

1. (10 pts.) Sketch the set of all the points z satisfying the equation:

$$|2\overline{z} + 4i| = 6$$

2. (10 pts.) Determine whether or not there exists an entire function $f(z)$ satisfying the following conditions:

$$|f(0)| = 1 \text{and } |f(z)| = 1/2 \text{for all } z \text{with } |z| = 1$$

If there is, give an example. If not, explain why.

3. (40 pts.; 10 pts. per each part) Let C be the circle of radius 4 centered at the origin (with the counter-clockwise direction). Compute the following integrals.

(a) $\displaystyle\int_C \overline{z} dz;$

(b) $\displaystyle\int_C \frac{\cosh(z)}{(z-5)(z+6i)}\, dz;$

(c) $\displaystyle\int_C \frac{z^4}{(z-i)^3}\, dz;$

(d) $\displaystyle\int_C \frac{e^z}{(z-i)(z-2i)}\, dz.$

4. (30 pts.; 10 pts. per each part) Find the Laurent series of the function

$$f(z) = \frac{1}{(z-1)(z-2)}$$

in the following regions:

(a) $|z| < 1;$

(b) $1 < |z| < 2;$

(c) $|z| > 2.$

5. (10 pts.) Find the linear fractional transformation T which maps the points $-1, 0, 1$ onto $-i, 1, i$ respectively.