MASTER'S EXAM - TOPOLOGY - JANUARY 1998

1. Prove that X is Hausdorff if and only if the diagonal $\Delta = \{(x, x) | x \in X\}$ is closed in $X \times X$.

2. Let $\{C_n\}$ be a sequence of closed, non-empty subsets of a compact space X such that $C_n \supseteq C_{n+1}$ for all n. Prove from the definition of compactness that $\cap C_n \neq \emptyset$.

3. Let U be a connected, non-empty open subset of $\mathbf{R}^2$. Prove that U is path-connected.

4. Let $f : X \to Y$ be a continuous bijection which is a closed map. Prove that f is a homeomorphism.

5. Prove that every separable metric space satisfies the second axiom of countability.