MASTERS COMPREHENSIVE EXAM—COMPLEX VARIABLES—January 2000

1. Indicate whether the given function is analytic on the set where it is defined. No proof is required.
   (i) $\cos z - 5z$
   (ii) $|z|^4$
   (iii) $\cos(\bar{z})$
   (iv) $\operatorname{Arg}(z)$ (the principal value of the argument function)
   (v) $\sin\left(z^3 - e^{2z}\right)$

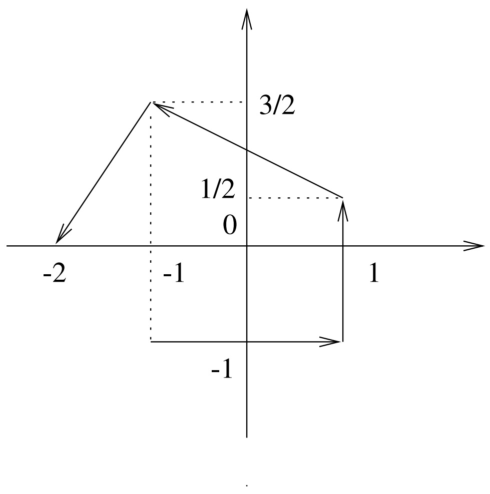
2. Let T be the transformation defined by

$$T(z) = \frac{z + \frac{1}{2}}{1 + \frac{1}{2}z}.$$

   Give a geometric description of the region in the complex plane determined by

$$\frac{1}{2} < |T(z)| < 1.$$

3. If C is the following contour from $-1 - i$ to -2, find the contour integral $\int_C z\, dz$.



4. Define

$$f(z) = \sum_{n=1}^{\infty} \frac{1}{5^n n! z^n} + \sum_{n=0}^{\infty} \frac{z^n}{3n^2 + n!}.$$

   Given that the series converge for all $z \neq 0$, find $\int_C f(z)\, dz$ if C is the unit circle taken counterclockwise.

5. Use the Residue Theorem to compute with justification the following integral:

$$\int_0^{\infty} \frac{\cos x}{x^2 + 1}\, dx.$$