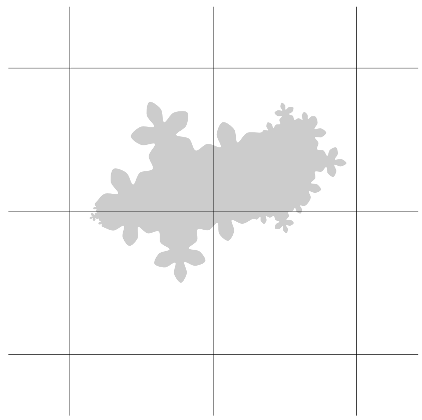
1. Find all integers of the form $n^4 + 4$ which are prime numbers.

   Hint: $n^4 + ? + 4$ is a perfect square.

(Continue on Problem 1)

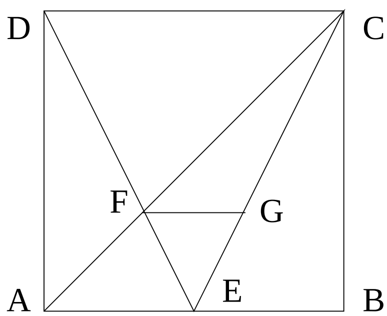
2. Suppose that S is a subset of the plane, like the shaded area in the figure below, and the area of S is greater than 1. Show that there exist two points $p = (x_1, y_1)$ and $q = (x_2, y_3)$ in S such that $x_1 - x_2$ and $y_1 - y_2$ are both integers.

   Hint: Cut the plane into a grid of squares of side length 1, as in the figure. Now imagine stacking the pieces on top of each other...



(Continue on Problem 2)

3. The square $\square ABCD$ has sides of length 2. Point E lies on the center of edge AB. Point F is the intersection of lines AC and DE. Line FG is parallel to line AB. Find the area of triangle ΔEFG.



(Continue on Problem 3)

4. Let

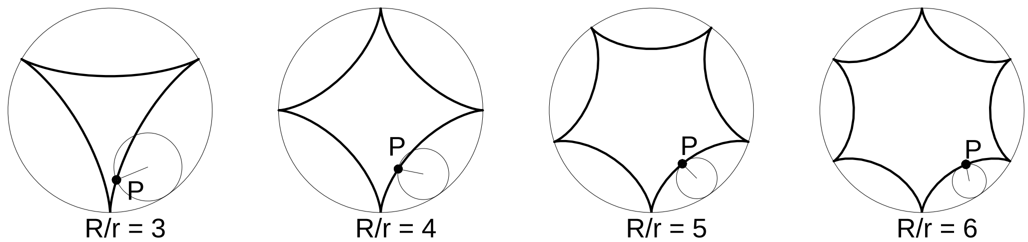
$$x = \sqrt[3]{1 + \sqrt[3]{1 + \sqrt[3]{1 + \sqrt[3]{1 + \cdots}}}}$$

Show that $1 < x < 2$.

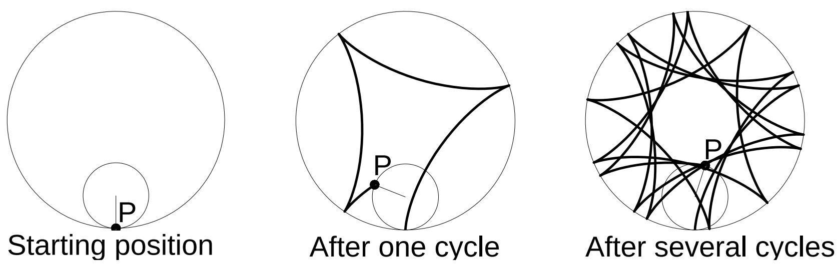
(Continue on Problem 4)

5. When a small circle of radius r rolls around (no sliding) the inside of a large circle of radius R, the trace of a point on the small circle is called a *hypocycloid*. We assume that both R and r are integers.

When the ratio R/r is an integer, P will return to its starting position after one full cycle of rolling around the large circle. In the diagrams below, we illustrate the trace of P with $R/r = 3$, 4, 5, 6, in thick dark curves.



When R/r is not an integer, P will not return to its starting position after one full rolling cycle. However, we can keep rotating the small circle and the trace of P is illustrated in below.



In this case, will P ever be able to go back to its starting position? If your answer is no, explain why. If your answer is yes, calculate the total number of full cycles (around the large circle) needed for P to go back to its starting position.

(Continue on Problem 5)

(Extra working space)

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