Potential Theory and Applications

MATH 6290

Time and Place: MW 11:30-12:45 p.m. in MSCS 509
Professor: Igor E. Pritsker
Office: MSCS 524
Office Hours: MWF 9:30-10:30 a.m.
Office Phone: 744-8220
E-mail: igor@math.okstate.edu
Web: http://www.math.okstate.edu/~igor/
Textbook: T. Ransford, Potential Theory in the Complex Plane, Cambridge University Press, 1995.

Potential theory has grown out of electrostatics and gravity theory. Distributions of charges on a conductor give great insight and illustration to many results in potential theory. It allows to give precise answers to many classical problems such as the Dirichlet problem, removable (erasable) sets for harmonic functions, etc. Further applications are found in approximation and interpolation of functions, in the theory of orthogonal polynomials, random matrices, number theory, and far beyond.

Brief contents

1. Harmonic functions: Analytic completion, series expansion, uniqueness principle, mean-value property, maximum-minimum principles, Poisson integral formula, boundary values, reflection principle, Harnack's inequality and theorem.
2. Subharmonic functions: Majorization by harmonic functions, mean-value inequality property, sequences of subharmonic functions, maximum principle, integrability.
3. Potentials: Maximum principle, continuity, weak* convergence of measures, energy problem, equilibrium measure, conductor potential, Frostman's theorem.
4. Capacity: Properties of capacity, transfinite diameter, Chebyshev constant, Evans' potential, generalized maximum principle, removable singularities of harmonic functions.
5. Dirichlet problem: Upper and lower functions, Perron's solution, regular boundary points, barrier, Green function, connection with conductor potential and conformal mapping, criteria of regularity, three-circle theorem, harmonic measure, two-constant theorem.
6. Applications: Inequalities for polynomials and analytic functions, zero distribution, etc.

Prerequisites: Complex and Real Analysis courses.

Additional references:
1. M. Tsuji, Potential Theory in Modern Function Theory, Maruzen, Tokyo, 1959.
2. W. K. Hayman and P. B. Kennedy, Subharmonic Functions, Academic Press, New York, 1976.
3. D. H. Armitage and S. J. Gardiner, Classical Potential Theory, Springer, New York, 2001.
4. L. L. Helms, Potential Theory, Springer, London, 2009.