Heegaard Floer Homology Exercise Set #2

Exercise 1: Let $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{w}$ be intersection points in $\mathbb{T}_\alpha \cap \mathbb{T}_\beta$, let $\phi_1 \in \pi_2(\mathbf{x}, \mathbf{y})$ and let $\phi_2 \in \pi_2(\mathbf{y}, \mathbf{w})$. Define $\phi_1 * \phi_2 \in \pi_2(\mathbf{x}, \mathbf{w})$ to be the homotopy class of the map formed by concatenation. Show that

1. $\mathcal{D}(\phi_1 * \phi_2) = \mathcal{D}(\phi_1) + \mathcal{D}(\phi_2)$, and
2. $\mu(\phi_1 * \phi_2) = \mu(\phi_1) + \mu(\phi_2)$.

Exercise 2: Compute the homology of each of the following chain complexes.

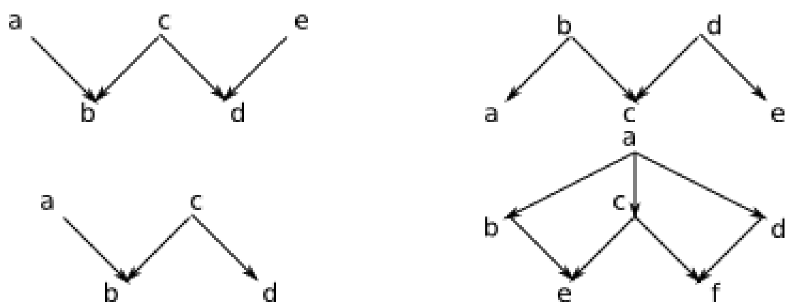

Figure 1: Possible $\widehat{CF}$ complexes

Exercise 3: Let Y be the example from lecture using the Heegaard diagram pictured below. In this exercise $\widehat{HF}(Y)$ will be computed.

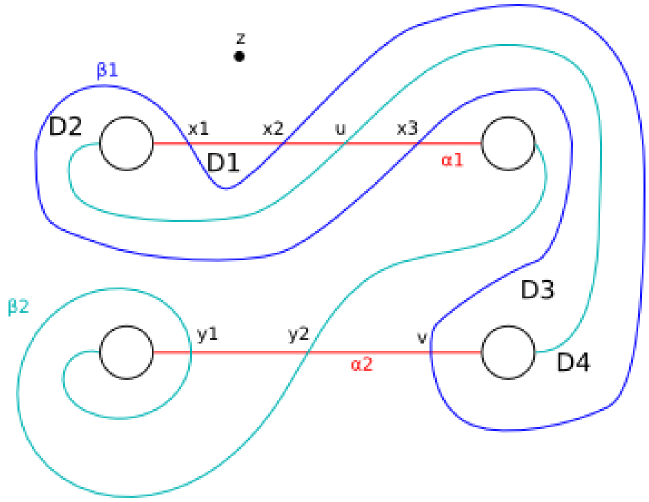

Figure 2: Heegaard diagram example

A) Show that $H_1(Y) \cong \mathbf{Z}/3\mathbf{Z}$. (*Hint: To start choose a nice set of generators for* $H_1(\Sigma)$)

B) Decide if each of the following domains is an image of a Whitney disk. If so compute the Maslov index for those Whitney disks. If the $\mu(\phi) = 1$, then decide which pairs of intersection points are connected by ϕ.

(i) D_3

(ii) D_4

(iii) $D_1 - D_4$

C) Compute the ϵ values for the following pairs of intersection points.

(i) $\epsilon(x_1y_1, x_1y_2)$

(ii) $\epsilon(x_1y_2, x_3y_1)$

(iii) $\epsilon(uv, x_3y_2)$

D) In addition to the contributing disks found in part (B), we found contributing Whitney disk between the following intersection points: $x_2y_1 \mapsto x_1y_1$, $x_2y_2 \mapsto x_1y_2$, and $x_2y_2 \mapsto x_3y_1$. Use all the information found to draw $\widehat{CF}(\Sigma, \underline{\alpha}, \underline{\beta}, z)$. (Hint: Based on part (A) there should be three equivalence classes of points.)

E) Compute $\widehat{HF}(Y)$.