Heegaard Floer Homology Exercise Set #3

Exercise 1: Show that the following is a short exact sequence of chain complexes.

$$0 \longrightarrow \widehat{CF}(\Sigma, \underline{\alpha}, \underline{\beta}, z, s) \xrightarrow{\iota} CF^+(\Sigma, \underline{\alpha}, \underline{\beta}, z, s) \xrightarrow{U} CF^+(\Sigma, \underline{\alpha}, \underline{\beta}, z, s) \longrightarrow 0$$

Here $\iota(x) = [x, 0]$.

Exercise 2: Recall that U is defined as an isomorphism of CF^∞. This restricts to a map U^- in CF^- and induces a map U^+ on CF^+. Furthermore, these maps induce U_*, U_*^-, and U_*^+ on their respective homologies. (In practice, abusing notation all of these maps are referred to as U.)

1. Which of these maps are always injective? ...surjective?
2. Recall the long exact sequence

   $$\cdots \longrightarrow HF^-(Y, s) \xrightarrow{i_*} HF^\infty(Y, s) \xrightarrow{\pi_*} HF^+(Y, s) \longrightarrow \cdots$$

   Show that $\ker((U_*^-)^k) = \ker(i_*)$ and that $\mathsf{im}((U_*^+)^k) = \mathsf{im}(\pi_*)$.
3. Prove that $HF^+_{red}(Y, s) \cong HF^-_{red}(Y, s)$.

Exercise 3: Prove that Y is an L-space if and only if $\mathsf{rk}\, HF^-(Y) = |H_1(Y; \mathbf{Z})|$.

Exercise 4: Consider the Y from last time (Heegaard diagram pictured below).

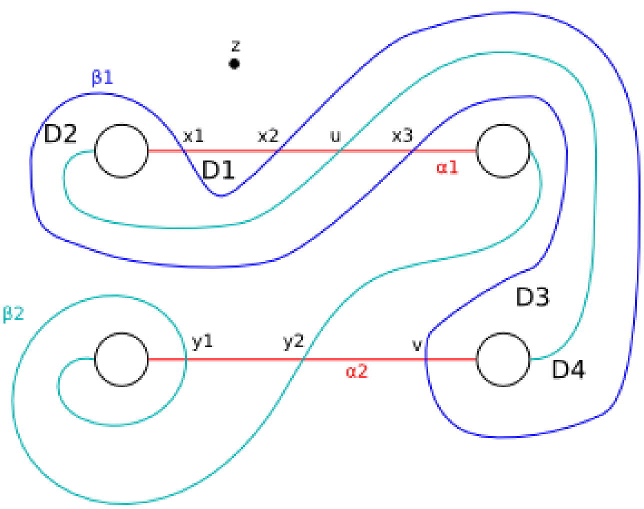

Figure 1: Heegaard diagram example

Let $s = s_z(x_1 y_1)$. Draw the complexes $CF^-(\Sigma, \underline{\alpha}, \underline{\beta}, z, s)$ and $CF^+(\Sigma, \underline{\alpha}, \underline{\beta}, z, s)$.