Exercise Set #3

Exercise 1: Two knots K and K' are *ambient isotopic* if the is a smooth isotopy $F_t : M \to M$ such that $F_0 = id_M$ and $F_1(K) = K'$.

(a) Prove that if $M = S^3$ then K is ambient isotopic to K' if and only if there is an orientation preserving automorphism $f : M \to M$ such that $f(K) = K'$.

(b) Give an example of two knots in a closed orientable manifold M that are equivalent but not ambient isotopic.

Exercise 2: Draw 2 different integral surgery descriptions of $L(7,3)$.

Exercise 3: Let K be a knot in S^3. Compute $H_1(S^3_{p/q}(K))$.

Exercise 4: Regard D^2 as $\{(x,y)|x^2+y^2 \leq 1\}$. Let $\varphi : D^2 \to D^2$ be rotation about the origin by $2\pi/n$, where n is a positive integer. Let E be a small disk centered at $(1/2, 0)$, small enough so that $E, \varphi(E), \dots, \varphi^{n-1}(E)$ are disjoint. Define

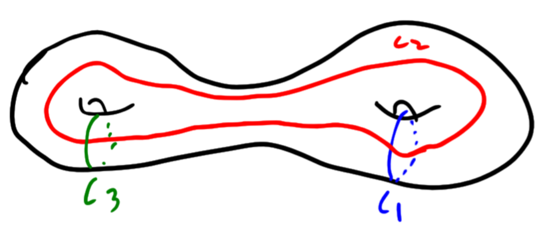
$$D^n := D^2 \setminus \bigcup_{i=0}^{n-1} \varphi^i(\mathsf{int}\, E),$$

and define

$$X_n := \frac{D_n \times I}{(x,0) \sim (\varphi(1), 1)}.$$

Describe X_n as a link exterior, and compute $\pi_1(X_n)$.

Exercise 5: Let H and H' be genus 2 handlebodies where h_1 is the gluing used in the standard genus 2 Heegaard decomposition of S^3 and $h_1 = h_2\tau_{c_3}\tau_{c_2}\tau_{c_1}$ with τ_{c_i} is a right-handed Dehn twist about the curve c_i (see below). Find a surgery description of $H \cup_{h_2} H'$.



Exercise 6: Prove that when $p/q = [x_1, \ldots, x_n]$, $L(p, q)$ has the following surgery description.