Exercise Set #4

Exercise 1: Show that a SFS of type $\mathbb{D}^2(2,2)$ is an I-bundle over the Klein bottle.

Exercise 2: Complete the proof that $\pi_1(S^2(2,2,2))$ is not abelian.

Exercise 3: Which SFS is the exterior of $T_{p,q}$, $M_{T_{p,q}}$, in S^3.

Exercise 4: Discuss the details of Moser's Theorem by following the steps below:

(i) Let $\langle \mu, \lambda \rangle$ be a basis for $\partial M_{T_{p,q}}$ and F denote the isotopy class of Seifert fibers in $\partial M_{T_{p,q}}$. Show that $F = pq\mu + \lambda$.

(ii) Let $\langle \mu', \lambda' \rangle$ be a basis for the boundary of the solid torus glued in via surgery for $S^3_{m/l}(T_{p,q})$. Find which isotopy class of curves is mapped to F.

(iii) Analyze the possible scenarios and complete the proof.

Exercise 5: Which lens spaces $L(m,l)$ are SFS's of type $S^2(2,2)$?

Exercise 6: Let K be a non-trivial knot and $N(K)$ denote its regular neighborhood. Let $h : V \to N(K)$ be a *faithful homeomorphism*, meaning that h takes the preferred meridian and longitude of V to the meridian and longitude of $N(K)$. We define the (p,q)-*cable* of K, $C_{p,q}$, to be $h(T_{p,q})$.

Show that the exterior of $C_{p,q}$ in $S^1 \times \mathbb{D}^2$ in a SFS of type $A^2(q)$.